Holographic dark energy in the DGP braneworld with GO cutoff

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We consider the holographic dark energy (HDE) model in the framework of DGP braneworld with Granda-Oliveros infrared (IR) cutoff, $L = (\alpha\dot{H} + \beta H^2)^{-1/2}$. With this choice for IR cutoff, we are able to derive evolution of the cosmological parameters such as the equation of state and the deceleration parameters, w and q, as the functions of the redshift parameter z. As far as we know, most previous models of HDE presented in the literatures, do not gives analytically $\omega = \omega(z)$ and $q = q(z)$. We plot the evolution of these parameters versus z and discuss that the results are compatible with the recent observations. With suitably choosing the parameters, this model can exhibit a transition from deceleration to the acceleration around $z \approx 0.6$. Then, we suggest a correspondence between the quintessence and tachyon scalar fields and HDE in the framework of DGP braneworld. This correspondence allows us to reconstruct the evolution of the scalar fields and the scalar potentials. We also investigate stability of the presented model by calculating the squared sound speed, v_s^2, whose sign determines the stability of the model. Our study shows that v_s^2 could be positive provided the parameters of the model are chosen suitably. In particular, for $\alpha > 1$, $\beta > 0$, and $\alpha < 1$, $\beta < 0$, we have $v_s^2 > 0$ during the history of the universe, and so the stable dark energy dominated universe can be achieved. This is in contrast to the HDE in standard cosmology, which is unstable against background perturbations and so cannot lead to a stable dark energy dominated universe.

I. INTRODUCTION

A complementary astrophysical data from type Ia Supernova, Large Scale Structure (LSS) and Cosmic Microwave Background (CMB) indicate that our Universe is currently undergoing a phase of accelerating [1]. A component which is responsible for this accelerated expansion is usually dubbed "dark energy" (DE). The simplest candidate for DE is the cosmological constant [2] which is located in the center both from theoretical and observational evidences. However, there are different alternative theories for the dynamical DE scenarios which have been proposed to interpret the accelerating universe. One of these models, which has arisen a lot of enthusiasm recently, is HDE. It was shown by Cohen et al. [3] that in quantum field theory a short distance cutoff could be related to a long distance cutoff (IR cutoff L) due to the limit sets by black hole formation. If the quantum zero-point energy density is due to a short distance cutoff, then the total energy in a region of size L should not exceed the mass of a black hole of the same size, namely $L^3\rho_D \leq LM_{pl}^2$. The largest L is the one saturating this inequality, so that we obtain the HDE density as [3]

$$\rho_D = 3c^2 M_{\rm pl}^2 L^{-2}, \tag{1}$$

where $M_{\rm pl}^2 = 8\pi G$ is the reduced Planck mass. In general the factor c^2 in holographic energy density can vary with time very slowly [4]. By slowly varying we mean that $\dot{(c^2)}/c^2$ is upper bounded by the Hubble expansion rate, H, i.e., [4]

$$\frac{\dot{(c^2)}}{c^2} \leq H. \tag{2}$$

It was also argued that c^2 depends on the IR length, L [4]. For the case of $L = H^{-1}$, one can take c^2 approximately constant in the late time where DE dominates ($\Omega_m < 1/3$) [4]. Since in the present work we consider the late time cosmology where the DE dominates, we shall assume the factor c^2 to be constant.

Depending on the IR cutoff, L, many HDE models have been proposed in the literatures. A comprehensive, but not complete, list of IR cutoffs which have been used includes the particle horizon radius $R_H \equiv a\int_0^t dt/a = a\int_0^a da/(Ha^2)$ [5, 6], the Hubble horizon $L = H^{-1}$ [7–9], the future event horizon radius $R_h \equiv a\int_t^\infty dt/a = a\int_a^\infty da/(Ha^2)$ [6], the apparent horizon radius $L = (H^2 + k/a^2)^{-1/2}$ [10], the Ricci scalar curvature radius $R_{CC} = (\dot{H} + H^2)^{-1/2}$ [11],

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the so-called Granda-Oliveros (GO) cutoff, which is the formal generalization of R_{CC}, namely $L = (\alpha\dot{H} + \beta H^2)^{-1/2}$ [12–14], the age of our Universe $T = \int_0^a da/(Ha)$ [15], the conformal age of our Universe $\eta \equiv \int_0^t dt/a = \int_0^a da/(a^2H)$ [16, 17], the radius of the cosmic null hypersurface [18], etc.

On the other side, in recent years, the theories of large extra dimensions in which the observed universe is realized as a brane embedded in a higher dimensional spacetime, have received a lot of interest. In these theories the cosmological evolution on the brane is described by an effective Friedmann equation that incorporates non-trivially with the effects of the bulk onto the brane. One of the well-know picture in the braneworld scenarios was proposed by Dvali-Gabadadze-Porrati (DGP) [19]. In this model our four-dimensional Universe is a Friedmann-Robertson-Walker (FRW) brane embedded in a five-dimensional Minkowskian bulk with infinite size. In this model the recovery of the usual gravitational laws on the brane is obtained by adding an Einstein-Hilbert term to the action of the brane computed with the brane intrinsic curvature. The self-accelerating branch of DGP model can explain the late time cosmic speed-up without recourse to DE or other components of energy [20, 21]. However, the self-accelerating DGP branch has ghost instabilities and it cannot realize phantom divide crossing by itself. To realize phantom divide crossing it is necessary to add at least a component of energy on the brane. On the other hand, the normal DGP branch cannot explain acceleration but it has the potential to realize a phantom-like phase by dynamical screening on the brane.

In the present work we consider the HDE model in the framework of DGP braneword with GO cutoff, $L = (\alpha\dot{H} + \beta H^2)^{-1/2}$, proposed in [13]. Our work differs from [13] in that, they studied HDE model with GO cutoff in standard cosmology, while we investigate this model in the framework of DGP braneword and incorporate the effect of the extra dimension on the evolution of the cosmological parameters on the brane. The main difference between the HDE with GO cutoff in the framework of DGP braneword, with the one considered in standard cosmology [13], is that the equation of state parameter of the HDE with GO cutoff in standard cosmology is a constant [13], namely

$$w_D = -1 + \frac{2(\alpha - 1)}{3\beta}, \tag{3}$$

however, as we shall see in DGP braneworld, due to the bulk effects, w_D becomes a time variable parameter. Clearly, a time variable DE is more compatible with observations. In particular, the analysis of data from WMAP9 or Planck-2013 results on CMB anisotropy, BAO distance ratios from recent galaxy surveys, magnitude-redshift relations for distant SNe Ia from SNLS3 and Union2.1 samples indicate that the time varying DE gives a better fit than a cosmological constant [22, 23]. In addition, current data still slightly favor the quintom DE scenario with EoS across the cosmological constant boundary $w_D = -1$ [22]. Furthermore, we are able to derive explicitly, the cosmological parameters on the brane as functions of redshift parameter and provide a profile of the cosmic evolution on the brane. Indeed, this is the advantages of the present model in compared to all other HDE models. As far as we know, this is the first model of HDE which leads to analytical solution, $w = w(z)$ and $q = q(z)$. Then, we establish the correspondence between our model and scalar field models of DE and reconstruct the evolution of scalar fields and potentials. Finally, we investigate the stability of this model against perturbation in different cases. We find that for some range of the parameter spaces our model is stable which indicates the viability of this model for explanation of the late time acceleration.

II. HDE IN DGP BRANEWORLD

We consider a homogeneous and isotropic FRW universe on the brane which is described by the line element

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right), \tag{4}$$

where $k = 0, 1, -1$ represent a flat, closed and open maximally symmetric space on the brane, respectively. The modified Friedmann equation in DGP braneworld model is given by [20]

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{\rho}{3M_{\rm pl}^2} + \frac{1}{4r_c^2}} + \frac{\epsilon}{2r_c}\right)^2, \tag{5}$$

where $\epsilon = \pm 1$ corresponds to the two branches of solutions [20], and $r_{\rm c}$ stands for crossover length scale between the small and large distances in DGP braneworld defined as [20]

$$r_c = \frac{M_{\rm pl}^2}{2M_5^3} = \frac{G_5}{2G_4}. \tag{6}$$

We also assume that there is no energy exchange between the brane and the bulk and so the energy conservation equation holds on the brane,

$$\dot{\rho} + 3H(1+w)\rho = 0. \tag{7}$$

For $r_c \gg 1$, the Friedmann equation in standard cosmology is recovered

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3M_{\rm pl}^2}. \tag{8}$$

Recent observations indicate that our universe is spatially flat. For a flat FRW universe on the brane, Eq. (5) reduces to

$$H^2 - \frac{\epsilon}{r_c}H = \frac{\rho}{3M_{\rm pl}^2}. \tag{9}$$

Depending on the sign of ϵ, there are two different branches for the DGP model. For $\epsilon = +1$ and in the absence of any kind of energy or matter field on the brane ($\rho = 0$), there is a de-Sitter solution for Eq. (9) with constant Hubble parameter

$$H = \frac{1}{r_c} \Rightarrow a(t) = a_0 e^{\frac{t}{r_c}}. \tag{10}$$

Clearly, Eq. (10) leads to an accelerating universe with constant equation of state parameter $\omega_D = -1$, exactly like the cosmological constant. However, there are some unsatisfactory problems with this solution. First of all, it suffers the well-known cosmological constant problems namely, the fine-tuning and the coincidence problems. Besides, it leads to a constant ω_D, while many cosmological evidences, especially the analysis of the type Ia supernova data indicates that the time varying DE gives a better fit than a cosmological constant [22–24]. Most of these data favor the evolution of the equation of state parameter and in particular it can have a transition from $\omega_D > -1$ to $\omega_D < -1$ at recent stage. Although some evidence such as the galaxy cluster gas mass fraction data do not support the time-varying ω_D [25], an overwhelming flood of papers has appeared to understand the $\omega_D = -1$ crossing in the past decade [26]. In addition, to arrive at Eq. (10) one ignores all parts of energy on the brane including DE, dark matter and byronic matter, which is not a reasonable assumption.

On the other hand, for $\epsilon = -1$ and $r_c \ll H^{-1}$, one can neglect the term H^2 in Eq. (9) to arrive at

$$H^2 = \frac{\rho^2}{36M_5^6}. \tag{11}$$

This is the Friedmann equation in spatially flat RS II braneworld [27]. Clearly, Eq. (11) does not have a self accelerating solution, so it implies the requirement of some kind of DE on the brane.

In the present paper, we consider the HDE model in flat DGP braneworld with GO cutoff, which is defined as [13]

$$L = (\alpha H^2 + \beta \dot{H})^{-1/2}. \tag{12}$$

With this IR cutoff, the energy density (1) can be written

$$\rho_D = 3M_{\rm pl}^2(\alpha H^2 + \beta \dot{H}), \tag{13}$$

where α and β are constants which should be constrained by the recent observational data and we have absorbed the constant c^2 in α and β. Hereafter, we work in a unit in which $M_{\rm pl}^2 = 1$. We also restrict our study to the current cosmological epoch, and hence we are not considering the contributions from matter and radiation by assuming that the dark energy ρ_D dominates, thus the Friedman equation becomes simpler. Substituting Eq. (13) into Friedmann equation (9) we can obtain the differential equation for the Hubble parameter as

$$H^2(1-\alpha) - \frac{\epsilon}{r_c}H - \beta\dot{H} = 0. \tag{14}$$

Solving this equation, the Hubble parameter is obtained as

$$H(t) = \frac{\epsilon}{r_{\rm c}(1-\alpha) + c_1 \epsilon e^{\frac{\epsilon t}{\beta r_c}}}, \tag{15}$$

where c_1 is constant of integration. Since for $r_{\rm c} \gg 1$, the effects of the extra dimension should be disappeared and the result of [13], namely

$$H(t) = \frac{\beta}{\alpha - 1}\frac{1}{t}, \tag{16}$$

must be restored, thus the constant c_1 should be chosen as

$$c_1 = \frac{r_{\rm c}(\alpha - 1)}{\epsilon}. \tag{17}$$

Substituting c_1 in Eq. (15), we obtain

$$H(t) = \frac{\epsilon}{r_{\rm c}(1-\alpha)(1-e^{\frac{\epsilon t}{\beta r_c}})}. \tag{18}$$

Taking the time derivative of Eq. (18) yields

$$\dot{H}(t) = \frac{1}{\beta r_{\rm c}^2(1-\alpha)}\frac{e^{\frac{\epsilon t}{\beta r_c}}}{(1-e^{\frac{\epsilon t}{\beta r_c}})^2}. \tag{19}$$

We can also solve Eq. (18) to obtain the scale factor. We find

$$a(t) = a_0\Big(e^{\frac{-\epsilon t}{\beta r_c}} - 1\Big)^{\frac{\beta}{\alpha-1}}. \tag{20}$$

Using the fact that $1+z = a_0/a$, where z is the redshift parameter, and combining Eqs. (18) and (20) we find explicitly the Hubble parameter as a function of z,

$$\begin{aligned} H(z) &= \frac{\epsilon}{r_{\rm c}(1-\alpha)}\frac{\Big[1+(1+z)^{\frac{1-\alpha}{\beta}}\Big]}{(1+z)^{\frac{1-\alpha}{\beta}}} \\ &= \frac{\epsilon}{r_{\rm c}(1-\alpha)}\Big[1+(1+z)^{\frac{\alpha-1}{\beta}}\Big]. \end{aligned} \tag{21}$$

This equation is valid in two cases. First, for $\epsilon = +1$ and $\alpha < 1$, and second for $\epsilon = -1$ and $\alpha > 1$, and has no solution for $\alpha = 1$. Inserting Eqs. (18) and (19) in (13), we get

$$\rho_D = \frac{3}{r_c^2(1-\alpha)^2}\frac{\alpha + (1-\alpha)e^{\frac{\epsilon t}{\beta r_c}}}{(1-e^{\frac{\epsilon t}{\beta r_c}})^2}. \tag{22}$$

Substituting Eqs. (18) and (19) in the time derivative of Eq. (13),

$$\dot{\rho}_D = 3(2\alpha\dot{H}H + \beta\ddot{H}), \tag{23}$$

we arrive at

$$\dot{\rho}_D = \frac{3\epsilon e^{\frac{\epsilon t}{\beta r_c}}}{\beta r_c^3(1-\alpha)^2(1-e^{\frac{\epsilon t}{\beta r_c}})^2}\Big[\frac{2(\alpha + (1-\alpha)e^{\frac{\epsilon t}{\beta r_c}})}{(1-e^{\frac{\epsilon t}{\beta r_c}})} + 1 - \alpha\Big]. \tag{24}$$

Combining Eqs. (18), (22) and (24) with (7), we find the equation of state parameter of HDE on the brane as a function of time,

$$\omega_D(t) = -1 - \frac{(1-\alpha)e^{\frac{\epsilon t}{\beta r_c}}}{3\beta}\Big[1 + \frac{1}{\alpha + (1-\alpha)e^{\frac{\epsilon t}{\beta r_c}}}\Big]. \tag{25}$$

In order to see the evolution of ω_D during the history of the universe, it is better to express it as a function of the redshift parameter z,

$$\omega_D(z) = -1 - \frac{1-\alpha}{3\beta}\Big[\frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}} + 2}{(1+(1+z)^{\frac{1-\alpha}{\beta}})(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})}\Big]. \tag{26}$$

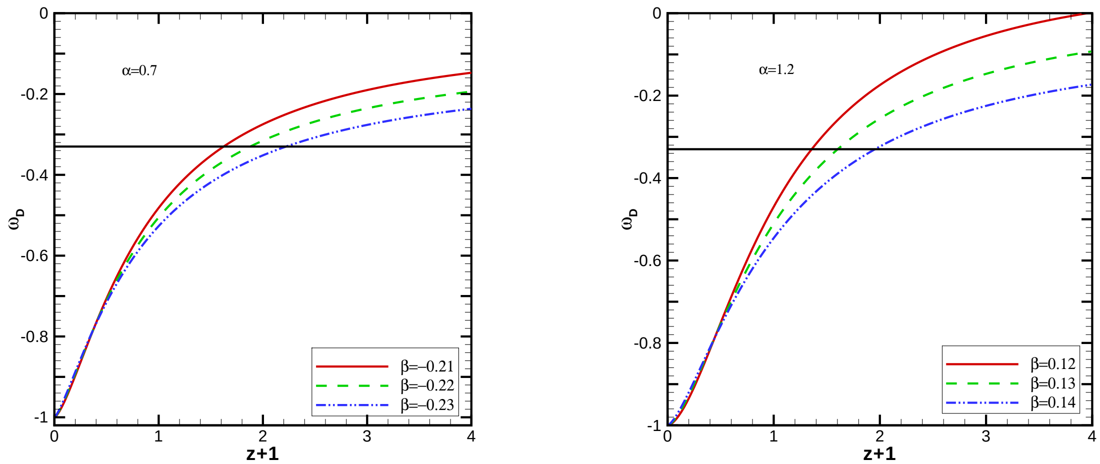

FIG. 1: The evolution of ω_D versus redshift parameter z for different model parameters. Left panel corresponds to $\alpha < 1$ and $\beta < 0$, while the right panel shows the case $\alpha > 1$ and $\beta > 0$.

To check the limit of (26) in standard cosmology where $r_c \gg 1$, we note that, from Eq. (20) and relation $a/a_0 = (1+z)^{-1}$ we have $(a/a_0)^{\frac{\alpha-1}{\beta}} = (1+z)^{\frac{1-\alpha}{\beta}} \to 0$, as $r_c \gg 1$. Therefore, (26) reduces to

$$\omega_D = -1 + \frac{2}{3}\frac{\alpha - 1}{\beta}, \tag{27}$$

which is exactly the result obtained in [13]. It is worth mentioning that unlike in standard cosmology, where the equation of state parameter of HDE with GO cutoff becomes a constant [13], in DGP braneworld scenario ω_D varies with time. Thus, it seems that the presence of the extra dimension brings rich physics. For $\alpha = 1$ we have $\omega_D = -1$, similar to the cosmological constant. Let us consider the case with $\alpha < 1$ and $\alpha > 1$ separately. In the first case where $\alpha < 1$, we find that the equation of state parameter can explain the acceleration of the universe provided $\beta < 0$. In the second case where $\alpha > 1$, the accelerated expansion can be achieved for $\beta > 0$. In both cases, our universe has a transition from deceleration to the acceleration phase around $0.4 \leq z \leq 1$, compatible with the recent observations [28–30], and mimics the cosmological constant at the late time. These results can be easily seen in Fig. 1 which shows the behaviour of ω_D versus z for both cases.

The first and second derivatives of the distance can be combined to obtain the deceleration parameter q. It was shown that the zero redshift value of q_0, is independent of space curvature, and can be obtained from the first and second derivatives of the coordinate distance [28]. It was argued that q_0, which indicates whether the universe is accelerating at the current epoch, can be obtained directly from the supernova and radio galaxy data [28]. The deceleration parameter is given by

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{28}$$

Using Eqs. (18) and (19), the deceleration parameter is obtained as

$$q = -1 - \frac{1-\alpha}{\beta\left[1 + (1+z)^{\frac{1-\alpha}{\beta}}\right]}. \tag{29}$$

For $\alpha = 1$ we have $q = -1$. Although, the equation of state and the deceleration parameters do not depend explicitly on the crossover length scale r_c which is the characterization of the DGP branworld, they depend on r_c via the relation between the scale factor and the redshift parameter as in (20). We have plotted the behavior of the deceleration parameter versus z in figure 2. Form this figures, we see that the transition from deceleration phase to the acceleration phase can be occurred around $0.4 \leq z \leq 1$, which is consistent with the recent observations [28]. The

zero-redshift value of the deceleration parameter is obtained as

$$q = -1 + \frac{\alpha - 1}{2\beta}. \tag{30}$$

Recent cosmological data from the combined sample of 192 supernovae and 30 radio galaxies, show that $q_0 = -0.30 \pm 0.18$, for a window function of width 0.4 in redshift and a transition from deceleration to the acceleration phase at redshift $z = 0.78 \pm 0.37$ [28]. The zero-redshift value (30), for $\alpha = 0.7$ and $\beta = -0.22$, leads to $q_0 = -0.32$, while for $\alpha = 1.2$ and $\beta = 0.15$, we get $q_0 = -0.33$, which is consistent with the observations [28].

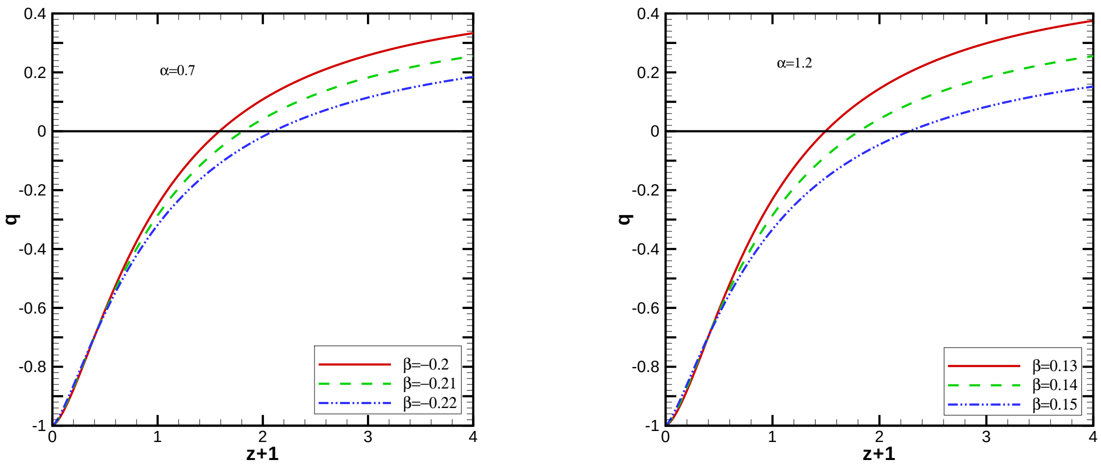

FIG. 2: The evolution of the deceleration parameter q versus redshift parameter z for different model parameters. Left panel corresponds to $\alpha < 1$ and $\beta < 0$, while the right panel shows the case $\alpha > 1$ and $\beta > 0$.

III. CORRESPONDENCE WITH SCALAR FIELD MODELS

The dynamical DE proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. Scalar fields naturally arise in particle physics including supersymmetric field theories and string/M theory. Therefore, scalar field is expected to reveal the dynamical mechanism and the nature of DE. However, although fundamental theories such as string/M theory do provide a number of possible candidates for scalar fields, they do not predict its potential, $V(\phi)$, uniquely. Consequently, it is meaningful to reconstruct the potential $V(\phi)$ from some DE models possessing some significant features of the quantum gravity theory, such as HDE models. Famous examples of scalar field DE models include quintessence [31], K-essence [32], tachyon [33], phantom [34], ghost condensate [35, 36], quintom [37], and so forth. For a comprehensive review on scalar filed models of dark energy, see [38, 39]. Generically, there are two points of view on the scalar field models of dynamical DE. One viewpoint regards the scalar field as a fundamental field of the nature. The nature of DE is, according to this viewpoint, completely attributed to some fundamental scalar field which is omnipresent in supersymmetric field theories and in string/M theory. The other viewpoint supports that the scalar field model is an effective description of an underlying theory of DE. If we regard the scalar field model as an effective description of such a theory, we should be capable of using the scalar field model to mimic the evolving behavior of the HDE and reconstructing the scalar field model according to the evolutionary behavior of HDE.

In this section we implement a correspondence between HDE in DGP braneworld with GO cutoff and various scalar field models, by equating the equation of state parameters for these models with the obtained equation of state parameter of Eq. (26).

A. Reconstructing holographic quintessence model

Let us start with reconstructing the potential and dynamics of quintessence scaler field. The energy density and pressure of the quintessence scalar field are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{31}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{32}$$

Thus the potential and the kinetic energy term can be written as

$$V(\phi) = \frac{1-\omega_\phi}{2}\rho_\phi, \tag{33}$$

$$\dot{\phi}^2 = (1+\omega_\phi)\rho_\phi. \tag{34}$$

where $\omega_\phi = p_\phi/\rho_\phi$. In order to implement the correspondence between HDE and quintessence scaler field, we identify $\rho_\phi = \rho_D$ and $\omega_\phi = \omega_D$. Using Eqs. (26), (33) and (34), we find

$$\dot{\phi}^2 = \frac{1}{\beta r_c^2(\alpha-1)} \frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}{(1+z)^{\frac{2(1-\alpha)}{\beta}}}, \tag{35}$$

$$V(z) = \frac{3}{2r_c^2}\frac{(1+z)^{\frac{2(\alpha-1)}{\beta}}}{(1-\alpha)^2}\Big[\Big(2\alpha(1+z)^{\frac{1-\alpha}{\beta}}+2\Big)\Big[(1+z)^{\frac{1-\alpha}{\beta}}+1\Big] + \frac{1-\alpha}{3\beta}\Big((1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2\Big)\Big]. \tag{36}$$

Eq. (35) can also be rewritten

$$\frac{d\phi}{d\ln a} = H^{-1}\dot{\phi} = \frac{1-\alpha}{\epsilon\sqrt{\beta(\alpha-1)}}\frac{\sqrt{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}}{1+(1+z)^{\frac{1-\alpha}{\beta}}}. \tag{37}$$

Using relation $a = a_0(1+z)^{-1}$ one can also obtain

$$\frac{d\phi}{dz} = \frac{1}{\epsilon(1+z)}\sqrt{\frac{\alpha-1}{\beta}}\frac{\sqrt{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}}{1+(1+z)^{\frac{1-\alpha}{\beta}}}. \tag{38}$$

Integrating, yields

$$\phi(z) = \int \frac{dz}{\epsilon(1+z)}\sqrt{\frac{\alpha-1}{\beta}}\frac{\sqrt{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}}{1+(1+z)^{\frac{1-\alpha}{\beta}}}. \tag{39}$$

Therefore, we have established holographic quintessence DE model and reconstructed the potential and the dynamics of scalar field according to evolutionary behaviour of HDE on the brane. Theoretically, one may expect to omit the redshift z from (36) and (39) to derive the potential as a function of the scalar field, namely, $V(\phi)$. However, due to the complexity of the equations, the analytical form of the potential $V = V(\phi)$ is hard to be derived. Nevertheless, we can plot the reconstructed potential as a function of ϕ numerically. The reconstructed quintessence potential $V(\phi)$ and the evolutionary form of the field are plotted in figure 3, where we have taken the zero redshift value of the scalar field equal to zero, namely, $\phi(z=0) = 0$. Selected curves are plotted for $\alpha < 1$ and the different model parameter $\beta < 0$. We can also plot the figures for the case $\alpha > 1$ and $\beta > 0$, however, the behaviour is the same as the former case and for the economic reason we have not plotted here the latter case. From these figures we can see the dynamics of the potential as well as the scalar field explicitly. Figure 3 shows that the reconstructed quintessence potential is steeper in the early epoch and becomes flat near the present time. In other words, it mimics a cosmological constant at the the present time.

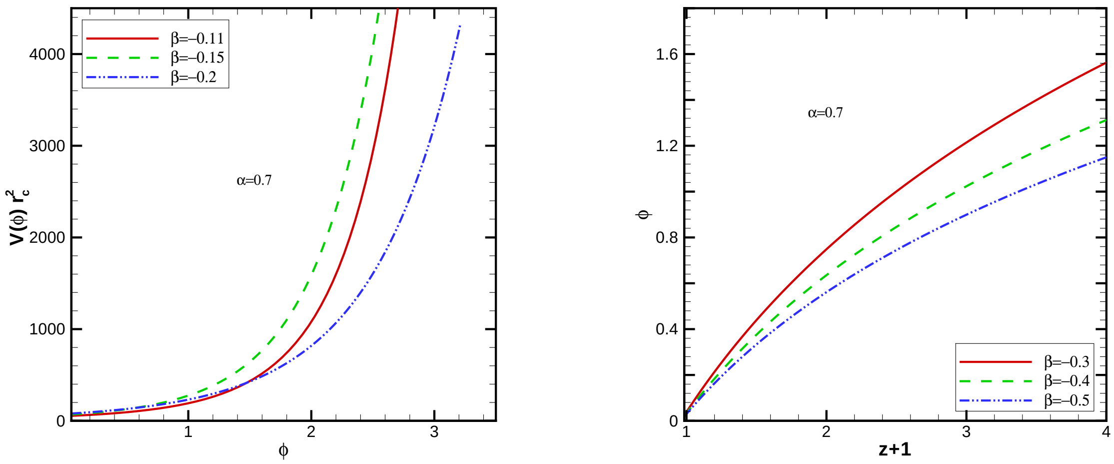

FIG. 3: Left panel shows the reconstruction of the quintessence potential $V(\phi)$ for different parameters. In the right panel the behavior of the quintessence scalar field $\phi(z)$ for HDE model in DGP braneworld is illustrated for different parameters.

B. Reconstructing holographic tachyon model

The tachyon field is another candidate for DE. The equation of state parameter of the rolling tachyon smoothly interpolates between -1 and 0 [40]. Thus, tachyon can be realized as a suitable candidate for the inflation at high energy [41] as well as a source of DE depending on the form of the tachyon potential [42]. Choosing different self-interacting potentials in the tachyon field model lead to different consequences for the resulting DE model. Due to all the above reasons, the reconstruction of tachyon potential $V(\phi)$ is of great importance. The correspondence between tachyon field and various DE scenarios such as HDE [43] and agegraphic DE [44] has been already established. The study has also been generalized to the entropy corrected holographic and agegraphic DE models [45].

The tachyon condensates in a class of string theories and can be described by an effective scalar field with a Lagrangian of the form by[46]

$$L = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}, \tag{40}$$

where $V(\phi)$ is the tachyon potential. The corresponding energy momentum tensor for the tachyon field can be written in a perfect fluid form

$$T_{\mu\nu} = (p+\rho)u_\mu u_\nu - p g_{\mu\nu} \tag{41}$$

where ρ and p are, respectively, the energy density and pressure of the tachyon, and the velocity u_μ is

$$u_\mu = \frac{\partial_\mu\phi}{\sqrt{\partial_\nu\phi\partial^\nu\phi}} \tag{42}$$

The energy density and pressure of tachyon field are given by

$$\rho \;=\; -T^0_0 = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{43}$$

$$p \;=\; T^i_i = V(\phi)\sqrt{(1-\dot{\phi}^2)}. \tag{44}$$

Thus the equation of state parameter of tachyon field is given by

$$\omega_T = \frac{p}{\rho} = \dot{\phi}^2 - 1. \tag{45}$$

To establish the correspondence between HDE and tachyon field, we equate ω_D with ω_T. Combining Eqs. (26), (43) and (44), we find

$$V(z) = \frac{3}{r_c^2(1-\alpha)^2}\sqrt{1-\frac{\alpha-1}{3\beta}\left[\frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}{(1+(1+z)^{\frac{1-\alpha}{\beta}})(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})}\right]}\,\frac{(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})(1+(1+z)^{\frac{1-\alpha}{\beta}})}{(1+z)^{\frac{2(1-\alpha)}{\beta}}}, \quad (46)$$

$$\dot{\phi}^2(z) = \frac{\alpha-1}{3\beta}\left[\frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}{(1+(1+z)^{\frac{1-\alpha}{\beta}})(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})}\right]. \quad (47)$$

We can further rewrite (47) as

$$\frac{d\phi}{d(\ln a)} = H^{-1}\dot{\phi} = \frac{-r_c(1+z)^{\frac{1-\alpha}{\beta}}}{\epsilon}\sqrt{\frac{(\alpha-1)^3}{3\beta}}\left[\frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}{(1+(1+z)^{\frac{1-\alpha}{\beta}})^3(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})}\right]^{\frac{1}{2}}. \quad (48)$$

Using the fact that

$$\frac{d\phi}{dz} = \frac{d\phi}{da}\frac{da}{dz} = -(1+z)^{-1}\frac{d\phi}{d(\ln a)}, \quad (49)$$

we have

$$\frac{d\phi}{dz} = \frac{r_c(1+z)^{\frac{1-\alpha}{\beta}}}{\epsilon(1+z)}\sqrt{\frac{(\alpha-1)^3}{3\beta}}\left[\frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}{(1+(1+z)^{\frac{1-\alpha}{\beta}})^3(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})}\right]^{\frac{1}{2}}. \quad (50)$$

Integrating and setting the constant of integration equal to zero by assuming $\phi(z=0)=0$, we obtain

$$\phi(z) = \int\frac{r_c(1+z)^{\frac{1-\alpha}{\beta}}}{\epsilon(1+z)}\sqrt{\frac{(\alpha-1)^3}{3\beta}}\left[\frac{(1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}+2}{(1+(1+z)^{\frac{1-\alpha}{\beta}})^3(1+\alpha(1+z)^{\frac{1-\alpha}{\beta}})}\right]^{\frac{1}{2}}dz. \quad (51)$$

In this way we connect the HDE in DGP braneworld with a tachyon field, and reconstruct the potential and the

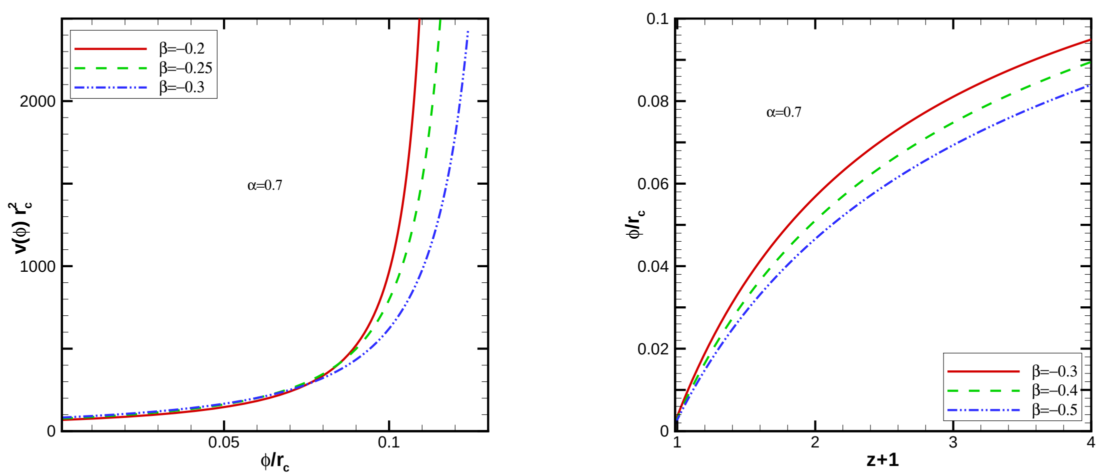

FIG. 4: Left panel shows the reconstruction of the tachyon potential $V(\phi)$ for different parameters. In the right panel the behavior of the tachyon field $\phi(z)$ for HDE model in DGP braneworld is illustrated for different parameters.

dynamics of the tachyon field which describe tachyon cosmology. The reconstructed tachyon field and the evolution of the tachyon potential are plotted in figure 4. Again, we see that tachyon potential is steeper in the early epoch and becomes flat near today. Thus, the universe enters a de-Sitter phase at the late time and the constant potential plays the role of cosmological constant.

IV. STABILITY OF THE MODEL

In this section we would like to investigate the stability of the HDE in DGP braneworld against perturbation. It is expected that any viable DE model should result a stable DE dominated universe. A simple, but not complete, approach to check the stability of a proposed DE model is to study the behavior of the square sound speed ($v_s^2 = dp/d\rho$) [47]. It was argued that the sign of v_s^2 plays a crucial role in determining the stability of the background evolution. If $v_s^2 < 0$, it implies that the perturbation of the background energy density is not an oscillatory function and may grow or decay with time, and so we have the classical instability of a given perturbation. On the other hand, the positivity of $v_s^2 > 0$ indicates that the perturbation in the energy density, propagates in the environment and so we expect a stable universe against perturbations. It is important to note that the positivity of v_s^2 is necessary but is not enough to conclude that the model is stable. Indeed, the negativity of it shows a sign of instability in the model. The behavior of the squared sound speed for HDE [48], agegraphic DE [49] and the ghost DE model [50, 51] were investigated. It was found that all these models [48–51] are instable against background perturbations and so cannot lead to a stable DE dominated universe.

In the linear perturbation regime, the perturbed energy density of the background can be written as

$$\rho(t,x) = \rho(t) + \delta\rho(t,x), \tag{52}$$

where $\rho(t)$ is unperturbed background energy density. The energy conservation equation ($\nabla_\mu T^{\mu\nu} = 0$) yields [47]

$$\delta\ddot{\rho} = v_s^2 \nabla^2 \delta\rho(t,x). \tag{53}$$

For $v_s^2 > 0$, Eq. (53) becomes a regular wave equation whose solution is given by $\delta\rho = \delta\rho_0 e^{-i\omega_0 t + ik.x}$, which indicates a propagation mode for the density perturbations. For $v_s^2 < 0$, the perturbation becomes an irregular wave equation and in this case the frequency of the oscillations are pure imaginary and the density perturbation will grow with time as $\delta\rho = \delta\rho_0 e^{\omega t + ik.x}$. Hence the negative squared speed shows an exponentially growing mode for a density perturbation. Thus the growing perturbation with time indicates a possible emergency of instabilities in the background. The

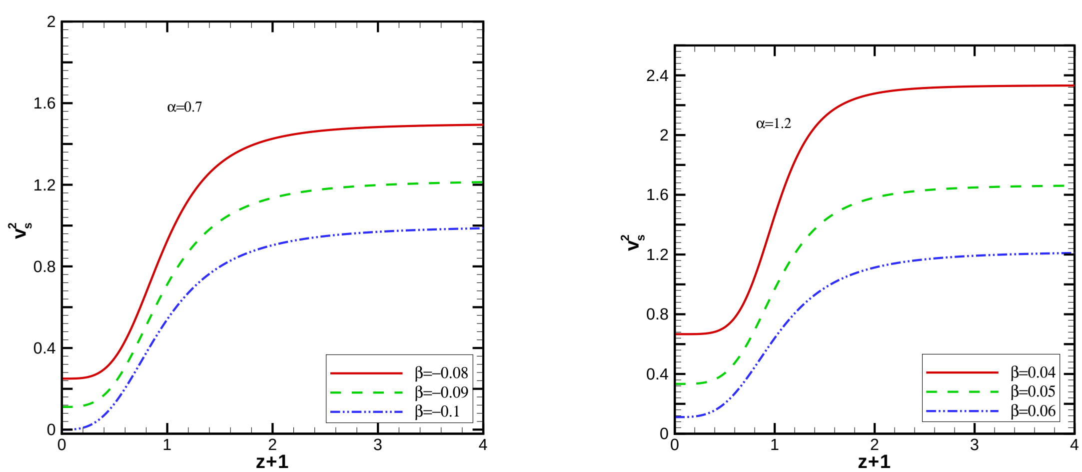

FIG. 5: The behaviour of v_s^2 for HDE model in DGP braneworld for different model parameters. Left panel corresponds to $\alpha < 1$ and $\beta < 0$, while the right panel shows the case $\alpha > 1$ and $\beta > 0$.

squared speed of sound can be written as

$$v_s^2 = \frac{dp}{d\rho} = \frac{\dot{p}}{\dot{\rho}}, \tag{54}$$

where

$$\dot{p} = \dot{\omega}_D \rho + \omega_D \dot{\rho}. \tag{55}$$

Taking the time derivative of (26) and substituting the result in (54), after some calculation, we arrive at

$$v_s^2 = \frac{\alpha - 1}{3\beta}\Big[\frac{4 + (1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}}{2 + (1+\alpha)(1+z)^{\frac{1-\alpha}{\beta}}}\Big] - 1. \tag{56}$$

The evolution of v_s^2 versus redshift parameter are plotted in figure 5. From these figures we see that v_s^2 can be positive provided the parameters of the model are chosen suitably. For example, for $\alpha < 1$, $\beta < 0$, and also for $\alpha > 1$, $\beta > 0$ the square of sound speed is always positive during the history of the universe, and so in these cases the stable DE dominated universe can be achieved.

V. CONCLUDING REMARKS

In this paper, we have investigated the holographic model of DE in the framework of DGP braneworld. We have chosen the GO IR cutoff in the form, $L = (\alpha\dot{H} + \beta H^2)^{-1/2}$, with two free parameters α and β, which should be constrained by comparing with observations. This cutoff is the generalization of the well-known Ricci scalar cutoff in flat universe. We have restricted our study to the current cosmological epoch, and so we have not considered the contributions from matter and radiation by assuming that the dark energy ρ_D dominates, thus the Friedman equation becomes simpler. The main difference between the HDE with GO cutoff studied in this work in the framework of DGP braneword, with the one considered in standard cosmology [13], is that the equation of state parameter of the HDE with GO cutoff in standard cosmology is a constant [13], however, in DGP braneworld, due to the bulk effects, w_D becomes a time variable parameter. The application of the HDE with GO cutoff, in DGP braneworld allows us to solve the Friedmann equation and derive the Hubble parameter, $H(z)$, as well as the scale factor, $a(t)$, analytically. We also obtained the equation of state and the deceleration parameters of HDE as the functions of the redshift parameter z and plotted the evolutionary behaviour of these parameters against z. The cosmological quantities depend on the two model parameter α and β. To show the viability of the model, we studied the zero-redshift values of the cosmological parameters by chosen the suitable values for α and β. For example, we found that for $\alpha = 0.7$ and $\beta = -0.21, -0.22, -0.23$, the transition from deceleration phase to the acceleration phase occurred around $0.4 \leq z \leq 1$, which is consistent with recent observations [28]. The zero-redshift value of the deceleration parameter was obtained $q_0 = -0.32$ for $\alpha = 0.7$, $\beta = -0.22$, and $q_0 = -0.33$ for $\alpha = 1.2$, $\beta = 0.15$ which is again compatible with the cosmological data [28]. Besides, for $\alpha = 1$ this model mimics a cosmological constant with $\omega_D = -1$, and $q = -1$, independent of the redshift parameter z. For $\alpha > 1$, $\beta > 0$ and $\alpha < 1$, $\beta < 0$ the equation of state parameter, however, is always larger than -1, and the Universe enters a de-Sitter phase at the late time.

We have also established a connection between the quintessence/tachyon scalar field and the HDE in DGP braneworld. As a result, we reconstructed the corresponding potentials of the scalar field, $V(\phi)$, and the dynamics of the scalar fields as a function of redshift parameter, $\phi = \phi(z)$, according to the evolutionary behavior of the HDE model. Finally, we studied the stability of the presented model by studying the evolution of the squared sound speed v_s^2 whose sign determines the sound stability of the model. Interestingly enough, we found that $v_s^2 > 0$ provided the parameters of the model are chosen suitably. For example, for $\alpha = 0.7$, $\beta = -0.1$, and $\alpha = 1.2$, $\beta = 0.06$ the squared sound speed is always positive during the history of the universe, and so in this case the stable DE dominated universe can be achieved. This is in contrast to HDE in standard cosmology, which is instable against background perturbations and so cannot lead to a stable DE dominated universe [48]. This implies that the presence of the extra dimension in HDE model, can bring rich physics and in particular it has an important effect on the stability of the HDE model. This issue deserves further investigations.

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